MC01 MOMENTUM, IMPULSE & 1D CONSERVATION

SPH4U

EQUATIONS

- Linear Momentum

$$\vec{p} = m\vec{v}$$

- Impulse

$$\Sigma\vec{F}\Delta t = \Delta\vec{p}$$

- 1D Conservation of Momentum

$$\Delta\vec{p}_1 = -\Delta\vec{p}_2$$
$$m_1\Delta\vec{v}_1 = m_2\Delta\vec{v}_2$$

MOMENTUM

- **Linear Momentum ($\overrightarrow{p}$) [N s = kg m/s]:** the product of the mass of a moving object and its velocity; a vector quantity

$$\vec{p} = m\vec{v}$$

  - m – mass of the object
  - $\vec{v}$ – velocity of the object
- Can be broken into vector components

$$p_x = mv_x \qquad p_y = mv_y$$

PROBLEM 1

Determine the momentum of a Pacific leatherback turtle of mass 8.6×10^2 kg, swimming at a velocity of 1.3 m/s [forward]. (The Pacific leatherback turtle is the world's largest species of turtle.)

PROBLEM 1 – SOLUTIONS

$m = 8.6 \times 10^2$ kg

$\vec{v} = 1.3$ m/s [forward]

$\vec{p} = ?$

$$\vec{p} = m\vec{v}$$

$$= (8.6 \times 10^2 \text{ kg})(1.3 \text{ m/s [forward]})$$

$$\vec{p} = 1.1 \times 10^3 \text{ kg·m/s [forward]}$$

The momentum of the turtle is 1.1×10^3 kg·m/s [forward].

IMPULSE

- Consider Newton's 2nd Law:

$$\Sigma\vec{F} = m\vec{a}$$

$$\Sigma\vec{F} = m\left(\frac{\Delta\vec{v}}{\Delta t}\right) = m\left(\frac{\vec{v}_f - \vec{v}_i}{\Delta t}\right)$$

$$\Sigma\vec{F}\Delta t = m\left(\vec{v}_f - \vec{v}_i\right) = m\vec{v}_f - m\vec{v}_i$$

$$\Sigma\vec{F}\Delta t = \vec{p}_f - \vec{p}_i$$

$$\boldsymbol{\Sigma\vec{F}\Delta t = \Delta\vec{p}}$$

IMPULSE – CONT.

- **Impulse ($\boldsymbol{\Delta \vec{p}}$) [N s = kg m/s]:** the product $\Sigma\vec{F}\Delta t$, equal to the object's <u>change in momentum</u>

$$\Sigma\vec{F}\Delta t = \Delta\vec{p}$$

- In components

$$\Sigma F_x \Delta t = \Delta p_x \qquad \Sigma F_y \Delta t = \Delta p_y$$

NEWTON'S 2ND LAW REVISITED

- We can now write Newton's 2nd Law in terms of impulse
- **Newton's 2nd Law:** the net force on an object equals the rate of change of the object's momentum.

$$\Sigma\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

- This form is more general than $\Sigma F = ma$, since we can now deal with changing mass.
- This is the original way Newton stated his 2nd law.
- NOTE: $\Sigma\vec{F}$ would be the average force in non-linear force applications

PROBLEM 2

A 57-g tennis ball is thrown upward and then struck just as it comes to rest at the top of its motion. The racket exerts an average horizontal force of magnitude 4.2×10^2 N on the tennis ball.

(a) Determine the speed of the ball after the collision if the average force is exerted on the ball for 4.5 ms.

(b) Repeat the calculation, assuming a time interval of 5.3 ms.

(c) Explain the meaning and advantage of follow-through in this example.

PROBLEM 2 – SOLUTIONS

(a) Since this is a one-dimensional problem, we can use components.

$m = 57\text{ g} = 0.057\text{ kg}$ $\qquad v_{ix} = 0$

$\sum F_x = 4.2 \times 10^2\text{ N}$ $\qquad v_{fx} = ?$

$\Delta t = 4.5\text{ ms} = 4.5 \times 10^{-3}\text{ s}$

$$\begin{aligned}
\sum F_x \Delta t &= \Delta p_x \\
\sum F_x \Delta t &= m(v_{fx} - v_{ix}) \\
\sum F_x \Delta t &= mv_{fx} - mv_{ix} \\
mv_{fx} &= \sum F_x \Delta t + mv_{ix} \\
v_{fx} &= \frac{\sum F_x \Delta t}{m} + v_{ix} \\
&= \frac{(4.2 \times 10^2\text{ N})(4.5 \times 10^{-3}\text{ s})}{0.057\text{ kg}} + 0 \\
v_{fx} &= 33\text{ m/s}
\end{aligned}$$

The speed of the tennis ball after the collision is 33 m/s.

PROBLEM 2 – SOLUTIONS CONT.

(b) $m = 57\text{ g} = 0.057\text{ kg}$ $\quad v_{ix} = 0$

$\sum F_x = 4.2 \times 10^2\text{ N}$ $\quad v_{fx} = ?$

$\Delta t = 5.3\text{ ms} = 5.3 \times 10^{-3}\text{ s}$

We use the same equation for v_{fx} as was derived in part (a):

$$v_{fx} = \frac{\sum F_x \Delta t}{m} + v_{ix}$$

$$= \frac{(4.2 \times 10^2\text{ N})(5.3 \times 10^{-3}\text{ s})}{0.057\text{ kg}} + 0$$

$$v_{fx} = 39\text{ m/s}$$

The speed of the tennis ball after the collision is 39 m/s.

PROBLEM 2 – SOLUTIONS CONT.

(c) The racket in (b) exerts the same average force as in (a) but over a longer time interval. The additional time interval of 0.8 ms is possible only if the player follows through in swinging the racket. The advantage of follow-through is that the final speed of the tennis ball after the collision is greater even though the average force on the ball is the same.

CONSERVATION OF MOMENTUM

- **Law of Conservation of Linear Momentum:** If the net force acting on a system of interacting objects is zero, then the linear momentum of the system before the interaction equals the linear momentum of the system after the interaction.

CONSERVATION OF MOMENTUM – CONT.

- From Newton's 3rd Law

$$\vec{F}_{12} = -\vec{F}_{21}$$
$$m_1\vec{a}_1 = -m_2\vec{a}_2$$
$$m_1\frac{\Delta\vec{v}_1}{\Delta t_1} = -m_2\frac{\Delta\vec{v}_2}{\Delta t_2}$$

- Since we are looking at the same time interval, $\Delta t_1 = \Delta t_2$, so we get

$$m_1\Delta\vec{v}_1 = -m_2\Delta\vec{v}_2$$

- Since $\Delta\vec{p} = m\Delta\vec{v}$, we get

$$\Delta\vec{p}_1 = -\Delta\vec{p}_2$$

CONSERVATION OF MOMENTUM – CONT.

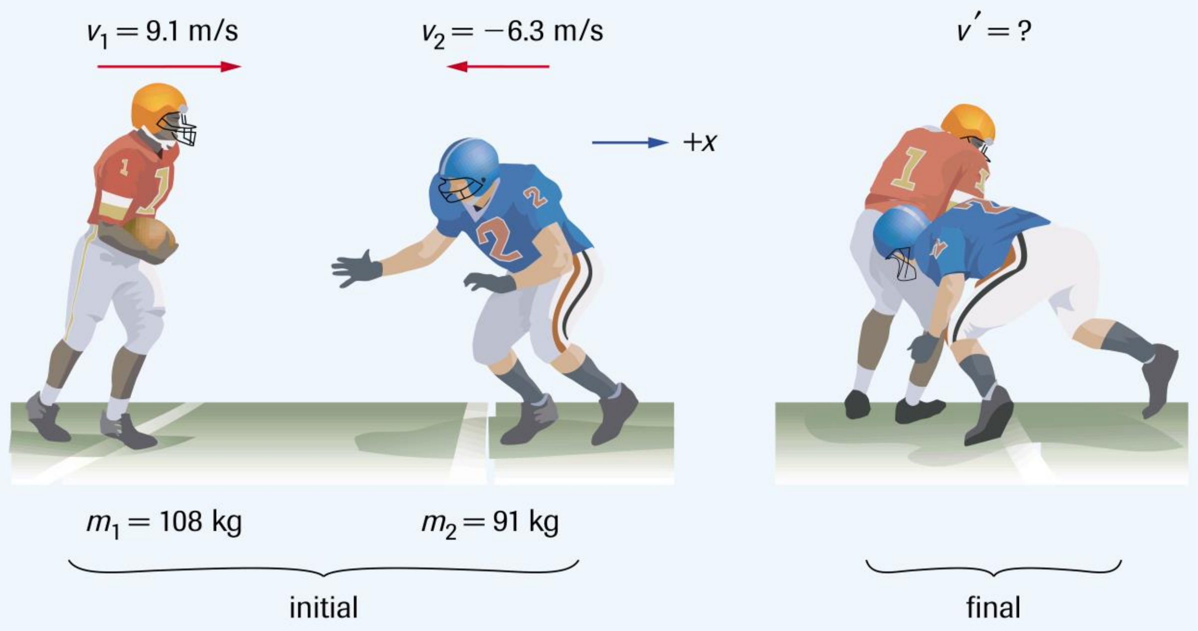
- Let’s consider what happens before and after the collision
  - Use prime (’) to denote after the collision

$$m_1\Delta\vec{v}_1 = -m_2\Delta\vec{v}_2$$
$$m_1(\vec{v}_1' - \vec{v}_1) = -m_2(\vec{v}_2' - \vec{v}_2)$$
$$m_1\vec{v}_1' - m_1\vec{v}_1 = -m_2\vec{v}_2' + m_2\vec{v}_2$$
$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$$
$$\vec{p}_{sys} = \vec{p}_{sys}'$$

- NOTE: you can break up these equations for component vectors, as well

PROBLEM 3

During a football game, a fullback of mass 108 kg, running at a speed of 9.1 m/s, is tackled head-on by a defensive back of mass 91 kg, running at a speed of 6.3 m/s. What is the speed of this pair just after the collision?



PROBLEM 3 – SOLUTIONS

During the collision, there is no net force on the two-player system. (The horizontal force exerted between the players is much larger than friction, which can therefore be neglected. In the vertical direction, there is no acceleration because there is no vertical net force.) Therefore, the momentum of this system is conserved. Thus,

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

where the subscript 1 refers to the fullback and the subscript 2 refers to the defensive back. Remember that v represents a velocity component, not a velocity magnitude.

PROBLEM 3 – SOLUTIONS CONT.

Since the two players have the same final velocity:

$$v_1' = v_2' = v'$$

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v'$$

$$v' = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

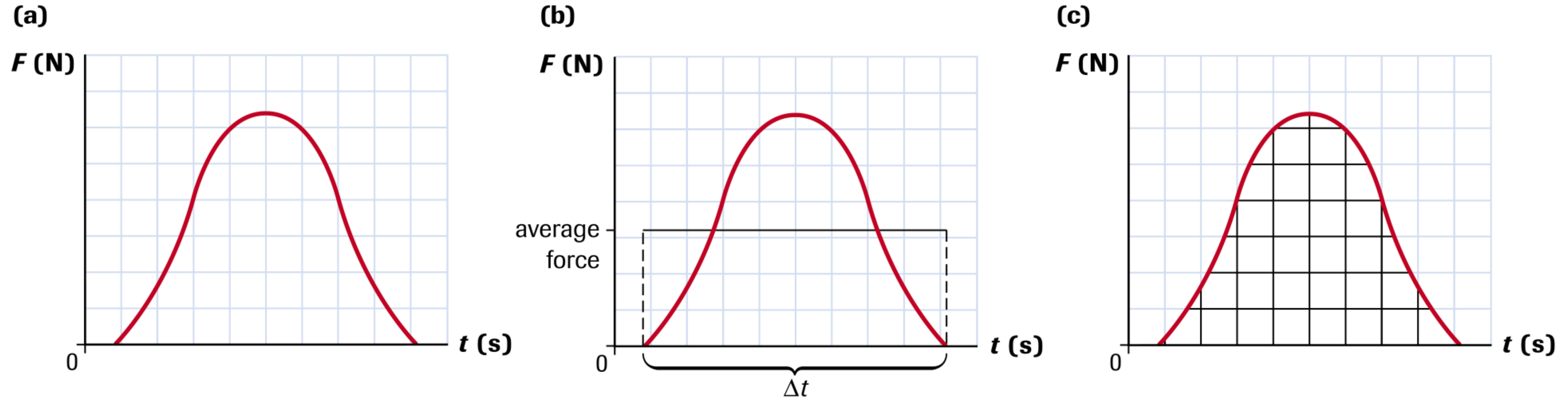
$$= \frac{(108 \text{ kg})(9.1 \text{ m/s}) + (91 \text{ kg})(-6.3 \text{ m/s})}{(108 \text{ kg} + 91 \text{ kg})}$$

$$v' = +2.1 \text{ m/s}$$

The final velocity of the players is 2.1 m/s in the direction of the initial velocity of the fullback (as the positive sign indicates).

FORCE-TIME GRAPHS

- Since $\Delta\vec{p} = \Sigma\vec{F}\Delta t$, we can find the change in momentum by finding the area under the curve of a force-time graph
- For non-linear applications of force, we estimate by counting the boxes



SUMMARY – MOMENTUM AND IMPULSE

- The linear momentum of an object is the product of the object's mass and velocity. It is a vector quantity whose SI base units are kg m/s.
- The impulse given to an object is the product of the average net force acting on the object and the time interval over which that force acts. It is a vector quantity whose SI base units are Ns.
- The impulse given to an object equals the change in momentum experienced by the object.

SUMMARY – CONSERVATION OF MOMENTUM IN 1 DIMENSION

- The law of conservation of linear momentum states that if the net force acting on a system is zero, then the momentum of the system is conserved.
- During an interaction between two objects in a system on which the total net force is zero, the change in momentum of one object is equal in magnitude, but opposite in direction, to the change in momentum of the other object.
- For any collision involving a system on which the total net force is zero, the total momentum before the collision equals the total momentum after the collision.

PRACTICE

Readings

- Section 5.1, pg 232
- Section 5.2, pg 239

Questions

- pg 238 #3,5,7,9,11
- pg 244 #1-3,5,7,9